First-principles study of superconductivity in high-pressure boron

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We study superconductivity in boron using first-principles LAPW calculations, the rigid-muffin-tin approximation, and the McMillan theory. Our results point to an electron-phonon mechanism producing transition temperatures near 10 K at high pressures, in agreement with recent measurements.

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I. INTRODUCTION

In a recent paper, Eremets et al. 1 reported experiments where boron transforms from a nonmetal at normal pressures to a superconductor at very high pressures above 160 GPa. They presented results that showed an increase of the superconducting temperature \( T_c \) from 6 K at 175 GPa to 11 K at 250 GPa.

Previous theoretical studies on the metallization of boron based on first-principles total-energy calculations 2 predicted that the 12-atom insulating form of boron \( (\alpha_{12}B) \) at atmospheric pressure transformed to a metallic body-centered tetragonal phase at 210 GPa and subsequently to an fcc structure at 360 GPa.

In this work we performed total-energy and band-structure calculations for the rhombohedral \( \alpha_{12}B \) and fcc phases. Our results for the fcc phase were used as an input to the rigid-muffin-tin (RMT) theory of Gaspari and Gyorffy 3 to determine the value of the Hopfield parameter \( \eta \) and subsequently evaluate the coupling constant \( \lambda \) and \( T_c \). Our results for the parameter \( \eta \) give values comparable to those reported in the past 4 for metallic hydrogen at very high pressure. Assuming that the fcc phase is the real high-pressure structure in the experiments of Eremets et al., 1 this indicates that the simple RMT theory accounts for high-pressure superconductivity in boron.

II. THEORY AND RESULTS

In this work we have applied the RMT theory 3 to calculate the Hopfield parameter \( \eta \) given by the expression

\[
\eta = \frac{E_F}{\pi^2 N(E_F)} \sum_l 2(l+1) \sin^2(\delta_{l+1} - \delta_l) \frac{N_{1}N_{l+1}}{N_l^{(1)}N_{l+1}^{(1)}}, \tag{1}
\]

where \( \delta_l \) is the scattering phase shift at the Fermi energy \( E_F \) and angular momentum \( l \), \( N_l^{(1)} \) is the single-scatterer density of states, which, as defined in Ref. 3, is an integral involving radial wave functions. \( N(E_F) \) is the total density of states (DOS) at \( E_F \) and \( N_l \) are the angular momentum components of the DOS inside the muffin-tin spheres. Equation (1) is exact up to \( l=1 \), but for higher values of \( l \) it involves non-spherical corrections. The necessary input to Eq. (1) was generated from a set of full potential linearized augmented plane-wave (LAPW) calculations that we performed for fcc B using touching muffin-tin (MT) spheres, and the tetrahedron method for the DOS.

We also performed an LAPW calculation for \( \alpha_{12}B \). This is a rhombohedral structure, space group \( R3m-D_{3h}^5 \) (No. 166), with measured lattice parameters 5 \( a=b=c=9.56 \) a.u. and angles \( \alpha=\beta=\gamma=58.06^\circ \). The energy bands of \( \alpha_{12}B \) are shown in Fig. 1. We obtain a valence bandwidth of 15.9 eV and an energy gap of 1.47 eV separating valence and conduction bands. The angular momentum character of the states can be seen from Fig. 2, which shows the expected dominance of the \( p \) states through the whole spectrum.

For fcc B we calculated an equilibrium lattice parameter \( a=5.37 \) a.u. and a bulk modulus of 282 GPa, to be compared with the values of 5.34 a.u. and 269 GPa reported by Mailhiot et al. 2 At \( a=4.60 \) a.u., we find that the pressure is 307 GPa, near the region where superconductivity is observed. 1 There we find a bulk modulus of 976 GPa, with shear moduli \( C_{11}-C_{12}=1101 \) GPa and \( C_{44}=134 \) GPa, showing that the fcc structure is at least metastable above this pressure. We note that for \( a=4.60 \) a.u. the nearest-neighbor B-B distance is 3.25 a.u., which is close to the B-B distance (3.37 a.u.) in the recently discovered superconductor MgB\(_2\). This leads us to speculate that superconductivity may have the same origin in both materials.

The band structure of fcc B confirms a metallic phase for all lattice parameters, with a rapidly increasing bandwidth going to higher pressures. The occupied bandwidth is 1.43 Ry at equilibrium and 1.88 Ry at \( a=4.60 \) a.u. The energy bands for \( a=4.60 \) a.u. are shown in Fig. 3. A fairly flat band appears near \( E_F \) in the \( XW \) direction at all lattice parameters. At \( X \) the band has strong \( s \) character, becoming \( p \)-like at \( W \). This situation is reminiscent of MgB\(_2\), where a similar flat band appears near \( E_F \) in the \( XW \) direction.
band just above $E_F$ has been identified as the origin of superconductivity in this compound.$^6$–$^9$ This band appears to be responsible for a peak in the DOS at $E_F$ shown in Fig. 4. We present the corresponding two-dimensional cross section of the Fermi surface in Fig. 5, showing hole pockets near $X$ and electron pockets near $W$. Also in Fig. 4 we find the $s$, $p$, and $d$ distributions of the DOS. In Fig. 4 we confirm that the strongest DOS component is that from the $p$ states. The values of DOS at $E_F$ enter Eq. (1) above for the evaluation of the Hopfield parameter $\eta$. The input and results using Eq. (1) at four lattice parameters are listed in Table I.

We note that in Table I the $l$ components of the DOS are given within the MT spheres as is the case in Fig. 4, and therefore they do not add up to the value of $N(E_F)$. The $p$ component $N_p(E_F)$ is, as expected, dominant and is about 60% of the value of the total inside the MT. The $p$-$d$ scattering, resulting from the $l=1$ term in Eq. (1), gives by far the largest contribution to the Hopfield parameter $\eta$. For the high-pressure case ($a=4.60$ a.u.), $\eta_{pd}=0.79\eta_{tot}$. The value of $\eta_{tot}$ is approximately the same as the value of $\eta=14.13$ eV/Å$^2$ previously reported$^4$ for metallic hydrogen at a higher pressure of 467 GPa. It is also interesting to note that the boron value of $\eta$ at equilibrium is very close to the value $\eta=7.627$ eV/Å$^2$ for Nb,$^1$ a typical transition-metal superconductor, where the $d$-$f$ scattering is the dominant term in Eq. (1).

To evaluate the critical temperature $T_c$ we have used the McMillan$^{11}$ approach defining an electron-phonon coupling constant $\lambda = \eta/M\langle \omega^2 \rangle$. Here $\eta$ is calculated from Eq. (1) as discussed above. For the average phonon frequency $\langle \omega \rangle$ we followed Eremets et al.$^1$ and chose a range of values from 1200 K to 1400 K. The resulting $\lambda$ is in the range 0.53 to 0.39, respectively. Continuing we use the McMillan equation

$$T_c = \frac{1.45\langle \omega \rangle}{\exp \left[ -\frac{1.04(1+\lambda)}{\lambda - \mu^* (1+0.62\lambda)} \right]}.$$  

We solve Eq. (2) for five values of the Coulomb pseudopotential $\mu^*=0.09$ to 0.13, and in the above range of $\omega$ and $\lambda$ values. The corresponding values of $T_c$ are shown in Fig. 6. Within the uncertainty of the values of the frequency $\omega$...
and near the value \( \omega = 1250 \) K, our model predicts \( T_c \) values in the range of the experiment that validates the assertion of an electron-phonon mechanism.

Finally, we want to comment on the pressure dependence of \( T_c \). Our calculations show that the parameter \( \eta \) goes from the value 7.82 eV/\( \text{Å}^2 \) at the equilibrium fcc volume to the value of 14.59 eV/\( \text{Å}^2 \) at a pressure of 307 GPa. It is important to note that \( N(E_F) \) follows the opposite trend, i.e., it reduces with increasing pressure. Assuming that the variation of \( \omega^v \) with pressure is not as strong as the variation of \( \eta \), we propose that the rapid increase of \( \eta \) is responsible for the observed increase of \( T_c \) with pressure.

In summary, we find that LAPW calculations, the RMT theory, and a McMillan analysis for \( T_c \) give a good description of superconductivity in boron at high pressures.

ACKNOWLEDGMENT

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**TABLE I.** Radius of the muffin-tin sphere \( R_s \), Fermi level \( E_F \), total DOS at \( E_F \), angular components of the DOS \( N_l \), free-scatterer DOS \( N_l^{(1)} \), scattering phase shifts \( \delta_l \), l components of the Hopfield parameter \( \eta \), and the total value of \( \eta \). The three columns are headed by the lattice parameters and the corresponding pressures.

<table>
<thead>
<tr>
<th>( a ) (a.u.)</th>
<th>( P ) (GPa)</th>
<th>( R_s ) (a.u.)</th>
<th>( E_F ) (Ry)</th>
<th>( N(E_F) ) [states/(Ry spin)]</th>
<th>( N_s(E_F) ) [states/(Ry spin)]</th>
<th>( N_p(E_F) ) [states/(Ry spin)]</th>
<th>( N_d(E_F) ) [states/(Ry spin)]</th>
<th>( N_s^{(1)} ) [states/(Ry spin)]</th>
<th>( N_p^{(1)} ) [states/(Ry spin)]</th>
<th>( N_d^{(1)} ) [states/(Ry spin)]</th>
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<td>1.776</td>
<td>1.050</td>
<td>0.114</td>
<td>0.469</td>
<td>0.164</td>
<td>0.187</td>
<td>0.818</td>
<td>0.010</td>
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<td>5.00</td>
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<td>1.768</td>
<td>1.408</td>
<td>1.328</td>
<td>0.172</td>
<td>0.568</td>
<td>0.193</td>
<td>0.231</td>
<td>1.058</td>
<td>0.105</td>
</tr>
<tr>
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<td>1.139</td>
<td>1.475</td>
<td>0.233</td>
<td>0.637</td>
<td>0.195</td>
<td>0.270</td>
<td>1.334</td>
<td>0.106</td>
</tr>
<tr>
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<td>2.121</td>
<td>0.784</td>
<td>1.514</td>
<td>0.276</td>
<td>0.769</td>
<td>0.155</td>
<td>0.324</td>
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