Effects of disorder on properties of \( A^{15} \) materials

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(Received 4 May 1982)

We have calculated the effects of disorder on the density of states, Fermi velocity, and Drude plasma frequency for \( V \) and its \( A^{15} \) compounds \( V_x X \), with \( X = \text{Al, Ga, Ge, Si, and Sn} \), and for \( \text{Nb} \) and its \( A^{15} \) compounds \( \text{Nb}_x X \), with \( X = \text{Al, Ga, Ge, Si, Sn, and Sb} \) using the electron-lifetime model and the results of band-structure calculations. In most cases the density of states and the superconducting transition temperature \( T_c \) are found to decrease with increasing disorder, in qualitative agreement with experiment. Exceptions are \( \text{Nb}_2 \text{Sb} \) and \( \text{Nb}_3 \text{Si} \), for which we have found a small increase in \( T_c \). We are also presenting calculations of the effects of disorder on the mean free path, BCS coherence length, London penetration depth, Ginzburg-Landau \( \kappa \), and the temperature dependence of the upper critical field for the above materials. Comparison with the existing experimental data is made.

I. INTRODUCTION

The \( A^{15} \) compounds include materials having the highest superconducting transition temperature (\( T_c \)) known. Many of the \( A^{15} \)'s also have very high critical fields and exhibit unusual properties for the electrical resistivity, magnetic susceptibility, and Knight shifts. As a result of the above properties and the desire to obtain even higher \( T_c \) and critical fields, these compounds have been extensively studied\(^1\) both theoretically and experimentally over the past 25 years.

Irradiation\(^2\) of superconducting materials has recently become a very active area of research due to the use of superconducting magnets in high-field applications. In applications such as bending magnets for beams in high-energy particle accelerators and magnetic confinement in proposed fusion reactors where the magnets must operate in an irradiation environment, the knowledge of the response of the critical properties of the superconductors to that irradiation must be known. Disorder induced by crystal growth conditions, \( \alpha \)-particle irradiation, and neutron damage yields a remarkable depression of \( T_c \) for the high-\( T_c \) superconducting \( A^{15} \) compounds \( \text{Nb}_2 \text{Sn}, V_x S\text{i} \), and similar materials. Testardi et al.\(^3\) have pointed out the universal nature of the depression of \( T_c \) by correlating it with the residual resistance ratio for various types of damage. A thorough investigation of neutron irradiation by Sweedler et al.\(^4\) demonstrated that the relative decrease of \( T_c \) in many \( A^{15} \) compounds followed a similar pattern as a function of the neutron flux. By contrast, disorder appears to have little influence on \( \text{Nb} \) (Ref. 3) that remains superconducting to \( T_c \approx 9 \text{ K} \) over a wide range of residual resistance. Further complication is provided by the disorder-enhanced superconducting temperature (from 1 to 5 K) in \( \text{Mo}_2 \text{Ge} \) (Ref. 4) which also exhibits the \( A^{15} \) structure.

The nature of the defects responsible for the depression of the superconducting temperature is not clear. On one hand, site-exchange disorder (replacement of \( A \) atoms by \( B \) atoms and vice versa in the \( A_3B \) structure) has been identified as an important defect in diffraction studies.\(^5\) On the other hand, an "unknown defect," perhaps in the form of atom displacement, which may be associated with the loss of translational symmetry has been suggested as a primary cause for the observable effects.\(^6\)

The effect of static disorder on the critical temperature \( T_c \) has been studied extensively in \( A^{15} \) compounds.\(^5\) However, the mechanism of the reduction of \( T_c \) in high-\( T_c \) \( A^{15} \) superconductors has remained a source of controversy. On the basis of the phonon-exchange mechanism of the superconducting electron pairing, it is natural to explore changes in the electron density of states (DOS) which may have profound influence on the transition temperature. This connection has been explored for a sharp DOS in a one-dimensional- (1D) type structure, which is easily distorted by disor-
Unfortunately, there is a great deal of uncertainty as to how the energy smearing of the DOS, \( N(E) \), will take place. The only microscopic theory is due to Labbé and Van Reuth\(^5\) but it is based on a special type of disorder and an oversimplified 1D model of an A15 compound. Some other explanations have been based on the theory of dirty superconductors with anisotropic gaps.\(^6\) However, as Gurvitch \textit{et al.}\(^6\) pointed out, the anisotropy necessary for explaining this reduction is unreasonably large. Appel\(^7\) has discussed the effect of long-range order on high \( T_c \). Another point of view has focused on the acoustic-plasmon mechanism,\(^8\) some arguments have been based on general smearing of \( N(E) \) with disorder,\(^9\) and finally a microscopic treatment\(^10\) of the consequences of both static and thermal disorder for superconductivity for a non-
constant DOS was presented using the generalized Elishberg gap equations.

The purpose of the present work is, by using the electron lifetime model introduced by Testardi and Mattheiss,\(^9\) to calculate \( N(E) \), the Fermi velocity \( v_F(E) \), the Drude plasma frequency \( \Omega_p(E) \), the mean free path \( l \), the \( T=0 \) BCS coherence length \( \xi_0 \), the \( T=0 \) London penetration length \( \lambda_L \), the Ginzburg-Landau \( \kappa \) near \( T_c \), and the temperature dependence of the upper critical field near \( T_c \) as a function of disorder broadening \( \Gamma \) or residual resistivity \( \rho_0 \) for \( V \), \( Nb \), and the A15 compounds \( Nb_3X \), \( V_3X \), with \( X=Al, Ga, Ge, Si, Sn \), and \( Nb_3Sb \). In applying this scheme, we have used the quantities \( N(E), v_F(E), \) and \( \Omega_p(E) \) of the perfectly ordered materials derived from the band-structure calculations of Boyer \textit{et al.}\(^11\) for \( V \) and \( Nb \) and of Klein \textit{et al.}\(^12,13\) for the A15 compounds. We wish to emphasize that a good quantitative understanding\(^13\) of the superconducting properties of the fully stoichiometric A15 compounds has been derived from these band-structure results without additional assumptions regarding quasi-one-dimensionality or resorting to model DOS singularities. A preliminary report of the present work has been given elsewhere.\(^14\)

### II. ELECTRON LIFETIME MODEL

As already discussed, the nature of the defect or defects responsible for the properties of A15 materials is not clear. Testardi and Mattheiss\(^9\) assumed that the defects (without specifying their exact nature) just broaden the DOS via the electron relaxation time \( \tau \), which decreases with disorder. If \( F(E, \Gamma=0) \) is a quantity corresponding to the perfectly ordered material whose energy dependence near the Fermi energy is known [here \( F(E, \Gamma=0) \) stands for \( N(E), v_F(E), \) or \( \Omega_p(E) \)], we can take the disorder effects into account with the convolution

\[
F(E, \Gamma) = \int S(E, E', \Gamma) F(E', \Gamma=0) dE',
\]

where \( S(E, E', \Gamma) \) is a broadening function that depends upon the electron damping \( \Gamma = \hbar/\tau \). The exact form of the broadening function is not too crucial when the broadening is large compared to any fine structure in \( F(E, \Gamma=0) \). All the calculations given below were done with a Lorentzian of half-width \( \Gamma = \hbar/\tau \) as a broadening function. The exact form is

\[
S(E, E', \Gamma) = \frac{\pi^{-1} \Gamma}{(E-E')^2 + \Gamma^2}.
\]

The electron lifetime \( \tau \) can be estimated from the Drude plasma frequency \( \Omega_p \) using the standard residual resistivity formula

\[
\rho_0 = \frac{4\pi\hbar}{\Omega_p^2} \frac{\Gamma}{\hbar^2} \frac{\Gamma}{\Omega_p^2 \tau}.
\]

Using this procedure, Testardi and Mattheiss\(^9\) calculated \( N(E), \Omega_p(E), \) and \( v_F(E) \) as a function of \( \Gamma \) or \( \rho_0 \) for \( V \), \( Nb \), \( V_3Si \), \( Nb_3Ge \), and \( Nb_3Sn \). Further assuming that the electron-phonon parameter \( \lambda \sim N(E_F) \), they found good agreement with experiment for the depression of \( T_c \) with disorder for \( V_3Si \) and \( Nb_3Sn \). We have made all the calculations given below with a Lorentzian broadening function while Testardi and Mattheiss\(^9\) used a thermal broadening function. To avoid complications from the Lorentzian wings in determining \( E_F \) for the high values of \( \Gamma \), one has to integrate the \( N(E, \Gamma) \) starting from much more negative values of \( E \) than for the \( \Gamma=0 \) case. We have also normalized Eq. (2) with \( \int S(E, E', \Gamma) dE' \) since we only do the \( E' \) integration on a finite-energy grid, and the integration of the Lorentzian is not exactly equal to one. Of course, one must also be careful to keep the total number of states \( Q \) in the energy band constant as we increase the electron width \( \Gamma \). The Fermi energy \( E_F(\Gamma) \) in the disordered state is determined from the constraint that the total number of states \( Q \) is constant, i.e.,

\[
Q = \int_{-\infty}^{E_F(\Gamma=0)} N(E, \Gamma=0) dE = \int_{-\infty}^{E_F(\Gamma)} N(E, \Gamma) dE.
\]

This is of course true for a defect that preserves the number of electrons in the crystal, e.g., disorder, as in our case, but is not true for, say, interstitials or vacancies. We have found for all the cases we examined, that \( E_F(\Gamma) \) increases with disorder.
III. RESULTS AND DISCUSSION

Using Eqs. (2) and (3) and the band-structure results\(^{11,12}\) of the perfectly ordered A15 compounds we have calculated \(N(E_F, \Gamma)\), \(v_F(E_F, \Gamma)\), and \(\Omega_p(E_F, \Gamma)\) as a function of electron damping \(\Gamma\), which is a measure of the disorder. Figures 1—3 show the calculated \(N(E_F, \Gamma)\), \(v_F(E_F, \Gamma)\), and \(\Omega_p(E_F, \Gamma)\) as a function of electron width \(\Gamma\). To obtain \(N(E_F)\), \(v_F\), and \(\Omega_p\) as a function of residual resistivity \(\rho_0\) or broadening temperature \(T_B\) (\(k_B\) is Boltzmann’s constant), one has to use the formulas that relate \(\rho_0\) to \(\Gamma\) and to \(T_B\). In particular, we have \(\Gamma\) given by

\[
\Gamma = \frac{\hbar}{\tau} = 1.34 \times 10^{-4}\Omega_p^2\rho_0, \quad (5)
\]

the broadening temperature \(T_B\) given by

\[
T_B = 1.554\Omega_p^2\rho_0, \quad (6)
\]

and the mean free path \(l\) given by

\[
l = v_F\tau = 4.91 \times 10^4 v_F^3/\Omega_p^2\rho_0, \quad (7)
\]

where \(v_F\) is in \(10^8\) cm/sec, \(l\) in Å, \(\rho_0\) in \(\mu\Omega\) cm and \(\hbar\Omega_p\) and \(\Gamma\) in eV. Equation (5) can be used to express \(N(E_F)\), \(\Omega_p\), and \(v_F\) as a function of \(\rho_0\).

We wish to point out that the information on Figs. 1—3 is enough to calculate, with the help of Eqs. (5)—(7), various properties (mean free path, \(T=0\) BCS coherence length, London penetration depth at Ginzburg-Landau \(\kappa\) near \(T_c\), and the temperature dependence of the upper critical field near \(T_c\)) of these materials as a function of \(\rho_0\). In particular, we have that the BCS coherence length at 0 K is given by\(^9\)

\[
\xi_0 = \frac{1.38 \times 10^4 v_F}{(1 + \lambda)T_c}, \quad (8)
\]

The London penetration depth at 0 K is given by

\[
\lambda_L = \frac{1.98 \times 10^4(1 + \lambda)^{1/2}}{\Omega_p}, \quad (9)
\]

The Ginzburg-Landau \(\kappa\) near \(T_c\) is

\[
\kappa = \frac{0.137(1 + \lambda)^{3/2}T_c}{\Omega_p v_F x(z)}, \quad (10)
\]

and the temperature dependence of the upper critical field near \(T_c\) is given by

\[
\frac{dH_c^2}{dT} \bigg|_{T_c} = \frac{3.18(1 + \lambda)^2 T_c}{\tau x(z)v_F}, \quad (11)
\]

where

\[
z = \frac{0.88\xi_0}{l} = \frac{0.246\Omega_p^2\rho_0}{(1 + \lambda)T_c}, \quad (12)
\]

\[
x(z) = \left[1 + \frac{z}{1.173}\right]^{-1}, \quad (13)
\]

where \(\xi_0, \lambda_L\) are in Å and \(dH_c^2/dT\) is in Oe/K.

Equations (10) and (11) do not include the strong-coupling corrections.\(^9\)

In Table I we show the values of the different quantities for the perfectly ordered A15 compounds. \(T_{c0}\) is the experimental value and \(\lambda^{\text{exp}}\) is the experimentally measured \(\lambda\) found by inverting McMillan’s equation. The rest of the parameters
are those derived from band-structure calculations.

Our results show that for all the A15 compounds we have studied except for Nb$_3$Sb and Nb$_3$Si, the DOS ultimately decreases as we increase the disorder. This seems to be expected for A15 compounds which are believed to exhibit a Fermi energy very near to a maximum of a sharp peak like V$_3$Si, Nb$_3$Al, Nb$_3$Sn, and V$_3$Ga. But we also find that A15 compounds exhibiting a Fermi energy not close to a peak in the DOS have a $N(E_F)$ which also decreases as we increase the disorder (V$_3$Al, V$_3$Ge, V$_3$Sn, and Nb$_3$Ge). Although in some cases, we have found a small increase of the $N(E_F)$ for low disorder further increase of $\Gamma$ always reduced $N(E_F)$. For the low-$T_c$ A15 Nb$_3$Sb we have a 20% increase in the DOS, while for Nb$_3$Si we also have a 15% increase. As we will discuss in more detail below, whatever was said above about $N(E_F)$ is applicable to the superconducting transition temperature $T_c$ under the assumption that the electron-phonon coupling parameter $\lambda$ is proportional to $N(E_F)$. This assumption seems to be reasonable and is supported by the calculations of Ref. 13. Using McMillan's equation one then gets $T_c$ as a function of $\lambda$ or $\rho_0$. While there seems to be a correlation between $\lambda$ or $T_c$ and $N(E_F)$ for the perfectly ordered materials, there is no such correlation be-
TABLE I. Measured superconducting temperature $T_c^{\text{exp}}$, experimentally derived value of the electron-phonon coupling $\lambda^{\text{exp}}$, and theoretical values at $E_F$ of the Fermi velocity, plasmon energy, and density of states (Ref. 12).

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c^{\text{exp}}$ (K)</th>
<th>$\lambda^{\text{exp}}$</th>
<th>$v_F(E_F, \Gamma=0)$ (10^6 cm/sec)</th>
<th>$\Omega_p(E_F, \Gamma=0)$ (eV)</th>
<th>$N(E_F\Gamma=0)$ (per Ry per cell)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>5.3</td>
<td>0.67</td>
<td>0.373</td>
<td>7.99</td>
<td>33.0</td>
</tr>
<tr>
<td>V$_3$Al</td>
<td>9.6</td>
<td>0.87</td>
<td>0.248</td>
<td>4.46</td>
<td>188.9</td>
</tr>
<tr>
<td>V$_3$Ga</td>
<td>16.5</td>
<td>1.17</td>
<td>0.205</td>
<td>4.63</td>
<td>295.6</td>
</tr>
<tr>
<td>V$_3$Si</td>
<td>17.1</td>
<td>1.12</td>
<td>0.210</td>
<td>4.02</td>
<td>200.2</td>
</tr>
<tr>
<td>V$_3$Ge</td>
<td>6.1</td>
<td>0.70</td>
<td>0.290</td>
<td>4.14</td>
<td>114.7</td>
</tr>
<tr>
<td>V$_3$Sn</td>
<td>3.8</td>
<td>0.61</td>
<td>0.270</td>
<td>3.73</td>
<td>123.0</td>
</tr>
<tr>
<td>Nb</td>
<td>9.2</td>
<td>0.94</td>
<td>0.565</td>
<td>9.45</td>
<td>26.3</td>
</tr>
<tr>
<td>Nb$_3$Al</td>
<td>18.6</td>
<td>1.60</td>
<td>0.277</td>
<td>4.58</td>
<td>199.2</td>
</tr>
<tr>
<td>Nb$_3$Ga</td>
<td>20.3</td>
<td>1.74</td>
<td>0.289</td>
<td>4.71</td>
<td>191.8</td>
</tr>
<tr>
<td>Nb$_3$Si</td>
<td>18.0</td>
<td>1.44</td>
<td>0.460</td>
<td>4.43</td>
<td>61.42</td>
</tr>
<tr>
<td>Nb$_3$Ge</td>
<td>23.2</td>
<td>1.80</td>
<td>0.356</td>
<td>4.35</td>
<td>106.7</td>
</tr>
<tr>
<td>Nb$_3$Sn</td>
<td>18.0</td>
<td>1.44</td>
<td>0.278</td>
<td>4.00</td>
<td>158.6</td>
</tr>
<tr>
<td>Nb$_3$Sb</td>
<td>0.2</td>
<td>0.38</td>
<td>0.451</td>
<td>3.74</td>
<td>51.95</td>
</tr>
</tbody>
</table>

between $T_c$ and the plasma frequency $\Omega_p(E_F)$ or the Fermi velocity $v_F$. There are relatively small variations of $\Omega_p$ and $v_F$ among the A15 materials. The introduction of disorder, as described above, has little effect on $\Omega_p(E_F)$. There is always a decrease of the order of 10—20%, except for the case of Nb$_3$Sb where there is a 10% increase. These results appear to be reasonable since with increasing disorder less free carriers exist and so $\Omega_p$ decreases. Our results for $\Omega_p$ could be tested with optical data on pure and disordered A15 materials, but unfortunately, no such experimental results are available. Such optical measurements together with resistivity data would provide an independent test of the electron damping as a function of resistivity for disordered A15 systems. Finally $v_F$ as a function of disorder $\Gamma$ seems to increase as $\Gamma$ increases for almost all the A15 compounds.

The superconducting transition temperature $T_c$ was obtained by assuming that $\lambda \sim N(E_F, \Gamma)$ and using McMillan’s equation with $\mu^* = 0.13$ and $\Theta_D$ to be constant. We used

$$\lambda = \frac{\lambda^{\text{exp}} N(E_F, \Gamma)}{N(E_F, \Gamma=0)},$$

where $\lambda^{\text{exp}}$ is the experimentally measured $\lambda$ for the perfectly ordered material found by inverting McMillan’s equation. Our results for $T_c/T_c^{\text{exp}}$ vs $\rho_0$ are shown in Figs. 4(a) and 4(b) for the Nb- and V-based materials, respectively. For the Nb A15's [Fig. 4(a)], we have found that $T_c$ for Nb$_3$Ga, Nb$_3$Sn, and Nb$_3$Al decrease by 80% when $\rho_0$ reaches 150—200 $\mu\Omega$ cm in qualitative agreement with experiment. Our detailed calculations for Nb$_3$Ge yield a smaller variation in $T_c$ than observed experimentally. Ruvalds and Soukoulis attributed the drop in $T_c$ in Nb$_3$Ge to overdamping of acoustic plasmons. On this point Klein et al. proposed an unusual phonon softening in Nb$_3$Ge in order to

FIG. 4. Calculated transition temperatures normalized to the transition temperature of the corresponding ordered material as a function of the residual resistivity $\rho_0$. 

reconcile the relatively low \( N(E_F) \) with the high \( T_c \). Recent tunneling experiments strongly support this idea. Hence, for Nb\(_3\)Ge, the drop in \( T_c \) seen experimentally is not coming from the decrease of \( N(E_F) \) but is probably a phonon effect. For Nb\(_3\)Si we predict that \( T_c \) will increase by 30\% as disorder increases, which may explain the disagreement between experiment and low-\( \lambda \) and -\( T_c \) values calculated at full stoichiometry. For the V-based Al\(_5\)'s [Fig. 4(b)], we have found that the \( T_c \) drop in V\(_3\)Si and V\(_3\)Ga is in qualitative agreement with experiment. The existing measured values of \( T_c \) for V\(_3\)Ge (Ref. 3) are also in qualitative agreement with our results, but due to difficulties in making clean samples there are no experimental values in the region with \( \rho_0 \leq 50 \, \mu\Omega \text{cm} \) to check if the calculated initial increase in \( T_c \) with \( \rho_0 \) is real. For V\(_3\)Sn and V\(_3\)Al, the decrease of \( T_c \) is 20\% and 60\%, respectively. Insufficient or no data for the latter compounds prohibits us from comparing our results with experiments.

Our results for the temperature dependence of the upper critical field near \( T_c \) are shown in Figs. 5(a) and 5(b). For all the Al\(_5\) compounds we have found that \( |dH_{c2}/dT| \) increases as the disorder increases, which is the opposite of what the transition temperature \( T_c \) does with disorder. This trend was also seen in experiments done on Nb\(_3\)Al, Nb\(_3\)Sn, V\(_3\)Si,\(^{14}\) and V\(_3\)Ga.\(^{19}\) We also want to mention that it is possible to get a higher upper critical field \( H_{c2} \) in the dirty limit \((\rho_0 > 0)\) than the clean limit \((\rho_0 = 0)\). This is so because the upper critical field is proportional to the product of the critical temperature \( T_c \) and \( dH_{c2}/dT \) as shown in the following equation:

\[
H_{c2}(0) \approx T_c \frac{dH_{c2}}{dT} \bigg|_{T_c}.
\]

Therefore it is possible to get a higher \( H_{c2} \) for \( \rho_0 > 0 \) provided that \( dH_{c2}/dT \) increases faster with \( \rho_0 \) than \( T_c \) decreases with \( \rho_0 \). This argument is correct only in the absence of Pauli paramagnetic limiting process. Figures 6(a) and 6(b) show the mean free path \( l \) as a function of disorder. As expected, \( l \) for all the Al\(_5\) compounds decreases with increasing disorder in a universal manner. In particular \( l \approx 125 - 150 \text{ \AA} \) for \( \rho_0 \approx 10 \, \mu\Omega \text{cm} \) and \( l \approx 5 - 7 \text{ \AA} \) for \( \rho_0 \approx 150 \, \mu\Omega \text{cm} \). As one can see from Figs. 7(a) and 7(b) the \( T = 0 \) BCS coherence length \( \xi_0 \) increases with disorder for almost all the Al\(_5\) compounds. The only exception is Nb\(_3\)Sb, but its anomalous behavior is related to its very low \( T_c \). By comparing Figs. 6 and 7 we see in the low disorder limit \((\rho_0 \approx 10 \, \mu\Omega \text{cm})\) none of the Al\(_5\)'s compounds satisfy the clean-limit approximation \( l \gg \xi_0 \). On the contrary \( \xi_0 \approx l \). On the other hand in the high disorder limit our results satisfy the dirty-limit approximation that \( \xi_0 \gg l \). Experimental results\(^{16}\) for Nb\(_3\)Sn and V\(_3\)Si for \( l \) and \( \xi_0 \) show qualitatively the same behavior as our calculations. The Ginzburg-
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Landau $\kappa$ near $T_c$ increases for all the A15's with disorder as seen in Fig. 8. The increase in $\kappa$ is almost an order of magnitude, provided that $\rho_0 \approx 180 \ \mu\Omega\text{cm}$. In Fig. 9 we plot the $T=0$ London penetration depth $\lambda_L$ versus disorder. Note that $\lambda_L$ decreases or increases by a maximum of 15%, and this reflects the changes in the plasma frequencies.

Finally, we note that our results for $V_3Si$, $Nb_3Ge$, and $Nb_3Sn$ are in at least qualitative agreement with those of Testardi and Mattheiss. Small differences between the calculations are due to self-consistency effects and different treatment of exchange in the band-structure calculations; and also due to the fact that we have used a Lorentzian broadening rather than the thermal broadening used by Testardi and Mattheiss.
IV. CONCLUSIONS

To summarize our findings, using the electronic band-structure calculations for the DOS, Fermi velocity, and plasma frequency for a series of A15 materials, we have calculated the $N(E_F)$, $\Omega_{p}(E_F)$, $v_F$, $dH_{c2}/dT$, $l$, $\xi_0$, $\kappa$, and $\lambda_L$ as a function of disorder. Disorder is introduced by averaging the above equations over an energy interval $\Gamma = \hbar / \tau$ around the Fermi level. We found that for the A15 materials studied (except for Nb$_3$Sb and Nb$_3$Si), $N(E_F)$ decreases as we increase disorder. By assuming that $\lambda \sim N(E_F)$, and using McMillan's equation for $T_c$ we find that $T_c$ always decreases (except for Nb$_3$Sb and Nb$_3$Si) with the introduction of disorder. Our detailed calculations suggest that the drop of $T_c$ in
the A15 superconductors is due to changes of the DOS, in particular the decrease of $N(E_F)$ with disorder. Magnetic susceptibility and specific-heat data on pure and disordered A15 compounds (especially for Nb$_3$Ge and V$_3$Al which do not have their $E_F$'s at a peak) will be useful for finding out changes in the DOS and whether or not a temperature-dependent susceptibility exists. The behavior of $T_c$ vs $\rho_0$ is in qualitative agreement with most of the existing experiments for $N(E_F)$ and $T_c$. Optical data on pure and disordered A15 materials in connection with resistivity data will give an independent estimate of the electron width $\Gamma$ as a function of resistivity.

We now wish to comment on the prospects of achieving higher superconductivity temperatures by shifting the Fermi energy to a favorable maximum in $N(E_F)$. This could be accomplished by alloying the A15 compounds and thus create A15 pseudobinarys. However, in this situation $N(E_F)$ may not reach a high value predicted by the rigid-band model but instead alloying can cause a reduction of the electron lifetime, and thus a reduction of $N(E_F)$. Another possibility may be to change the electronic structure by ion implantation of hydrogen in compounds like Nb$_3$Ge, Nb$_5$Si, and V$_3$Sn, with the goal of increasing $N(E_F)$, provided that the induced disorder is below a prescribed limit. However, the explorations for achieving a higher $N(E_F)$ need to be coupled with the phonon behavior, because it is possible that mechanisms which may enhance $N(E_F)$ could harden the phonon modes and therefore prevent the raising of $T_c$. Finally, it would also be interesting to calculate $N(E)$ for the disordered A15 compounds using the coherent-potential approximation and thus remove the uncertainties of the electron-lifetime model.

ACKNOWLEDGMENTS

We are indebted to W. E. Pickett for many useful discussions and suggestions. We also wish to thank B. M. Klein and D. U. Gubser for comments on the manuscript and L. N. Blohm for technical assistance.


